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The Theory of Cross Point Regions with Ideal Cross Point Regions for Medical Image Compression

DANG Thanh Tin, VU Dinh Thanh, NGUYEN Kim Sach, and Seihaku HIGUCHI †

Faculty of Electrical and Electronics Engineering, Hochiminh City University of Technology
268 Lythuongkiet, District 10, Hochiminh City, Vietnam
† Department of Information System Engineering, Faculty of Engineering, Osaka Sangyo University
3-1-1 Nakagaito Daitoshi, Osaka 574-8530, Japan
Email : dttin@hcmut.edu.vn

(Received August 24, 2006, in final form October 15, 2006)

Abstract : This paper presents the scheme LIC-ICR (Lossless Image Compression with Ideal Cross point Regions in the theory of cross point regions) for lossless encoding and decoding images, especially medical images with the optimization of probability of cross points which are neighbor to the points of grey levels 2^n. The base of this statement is the effect of Gray coding on cross points. Before Gray coding an adjacent data set of cross points is determined, it is called the ideal cross point region. After Gray coding this region always contains bits 1 on the bit plane $n-1$ (or bits 0 on the bit plane $n-2$) after bit plane decomposition. This is like the characteristic of images being that the data do not change much in a specific area, espescially in medical images which have many regions with the same grey levels. The only bit state in ideal cross point regions is very good at entropy coding because probabilities of data bits is 1 there. This optimization of probability has important effects on encoding and decoding processes of lossless medical image compression.

Key words : Gray code, cross point, cross point region, probability of bits, arithmetic coding

1. INTRODUCTION

This paper is a development of the papers [1], [2] that presented the definition of cross points and cross point regions, the propositions of bit states of cross points and the consequences about entropy of obtained data on cross point regions after Gray code transformation. It presents the effect of Gray coding on the calculating process of probability of bits in ideal cross point regions.

Cross points are neighbor points around the points of grey levels 2^n. The points of grey levels 2^n may or may not exist in data. The original data points whose values are less than 2^n have bit states being much different from those of the data points being greater than or equal to 2^n [3], [4]. The change of bit states for Gray code transformation is studied by many authors [5]–[8], ..., however the number of bits and the distribution of these bits of Gray codes in the ideal cross point regions are not mentioned yet. This leads to a new scheme for data compression. The data are arbitrary, but in ideal cross point regions the change of bit states are systematically determined after Gray coding, so the probability of data bits in ideal cross point regions is maximal.

This paper has six sections. After this introduction, the section 2 mentions the definitions of ideal cross point regions, the propositions about characteristics of Gray codes of cross points, this section also presents the bit states and the probability of bits in ideal cross point regions. Therefore, this section describes the theory used in the step "modelling" of entropy coding [8]. The coding algorithm in the entropy coding is Jones' [9], mentioned in the section 3. The section 4 introduces the scheme LIC-ICR (Lossless Image Compression with Ideal Cross point Regions in the theory of cross point regions). This scheme is an entropy coding with the ideal cross point regions used in the step "modelling". In the section 5, some our results obtained from using the theory above are presented in comparison with the other methods : Advanced Encryption Standard (WinRar 9.0) and lossless wavelet transform (JPEG 2000) ; and the section 6 is the conclusion and the scope for future researches.

2. IDEAL CROSS POINT REGIONS AND BIT STATES

2.1 Effect of Gray coding on data bits in ideal cross point regions on the bit plane $n-1$

DEFINITION 2.1. *Let the positive integer N be the bit length of data points, the region of cross points Ao (n), with n from 1 to (N − 1), is a set of data points whose grey values are from* $(2^n - 2^{n-1})$ *to* $(2^n + 2^{n-1}-1)$*. The point of grey value* 2^n *(if existing) is called the center point of the cross point region, and the grey value* 2^n *is called the central value. These regions are called the ideal cross point regions (ICRs) of type A..*

With Definition 2.1 data points in ICRs have grey values that satisfy the rule

$$VA\ (n) = \{2^n - 2^{n-1}, ..., 2^n + 2^{n-1}-1\}. \tag{1}$$

The value n is the exponent of central values 2^n, it is also used to determine the number of bit plane for data bit compression. The rule $V_A\ (n)$ can be divided into two groups : $V_{A\ l}\ (n)$ and $V_{A\ g}\ (n)$ with $V_{A\ l}\ (n) = \{2^n - 2^{n-1}, 2^n - 2^{n-1} + 1, ..., 2^n -2, 2^n -1\}$, and $V_{A\ g}\ (n) = \{2^n, 2^n+1, ..., 2^n + 2^{n-1}-2, 2^n + 2^{n-1}-1\}$.

For example, when $n = 3$, $V_A\ (n) = \{2^3 - 2^{3-1}, ..., 2^3 + 2^{3-1} -1\} = \{4, 5, 6, 7, 8, 9, 10, 11\}$.

PROPOSITION 2.1. *Let n be the exponent of the central value* 2^n *in the ideal cross point region* A_o *(n), n is from 1 to (N −1), N is a positive integer of bit length. Bits of Gray codes* *in the ideal cross point region $A_o(n)$ on the bit plane $n-1$ are bits 1.*

Proof of Proposition 2.1.

According to Definition 2.1, with a value of n in the interval $1\div(N-1)$, grey values of data points in the region $\boldsymbol{A_o(n)}$ that satisfy $V_{A\,l}\,(n)$ are expanded under the form of polynomials of radix 2 as the following

$$0.\,2^{N-1}+...+0.2^{n}+1.\,2^{n-1}+x.\,2^{n-2}+...+x.\,2^{0}. \tag{2}$$

Data points in the region $\boldsymbol{A_o(n)}$ that satisfy $V_{Ag}\,(n)$ are expanded by the following polynomial

$$0.2^{N-1}+...+0.2^{n+1}+1.2^{n}+0.2^{n-1}+x.2^{n-2}+...+x\,2^{0}, \tag{3}$$

where x are bits 1 or 0.

After Gray code transformation, (2) and (3) become (4) and (5) respectively

$$0.2^{N-1}+...+0.2^{n}+\mathbf{1.2^{n-1}}\ x.2^{n-2}+...+x.2^{0}, \tag{4}$$

$$0.2^{N-1}+...+0.2^{n+1}+1.2^{n}+\mathbf{1.2^{n-1}}+x.2^{n-2}+...+x.\,2^{0}. \tag{5}$$

By combining (4) and (5), the region $A_o(n)$ on the bit plane $n-1$ always contains bits 1.

For example, when $n = 3$, the central value is 2^3, $A_o(3) = \{4, 5, 6, 7, 8, 9, 10, 11\}$. After Gray coding Gray codes of these values are 6, 7, 5, 4, 12, 13, 15, 14 respectively, all of them have bits 1 on the bit plane 2 (= 3 − 1). This is very good to compress data because the probability of bit 1 in ICRs is always 1, and the probability of bit 0 in ICRs is always 0. This problem plays an important role to optimize the probability of data bits in the step of modeling of entropy coding.

CONSEQUENCE 2.1. *In the ideal cross point regions A_o, after Gray code transformation the entropy of obtained data on a certain bit plane is minimum.*

Proof of Consequence 2.1.

We can easily see that in the ideal cross point regions $A_o\,(n)$ on the bit plane $n-1$, before Gray code transformation, the probabilities of bit 1 and bit 0 are random, therefore the entropy of the corresponding bit string is often $H = -P(1).\log_2 P(1) - P(0).\log_2 P(0) > 0$.

After Gray code transformation, the ideal cross point regions contain all bits 1, so the probability of bit 1 in these regions is 1, and the probability of bit 0 is 0 there. This is the reason for that the entropy of this bit string is always $H = -P(1).\log_2 P(1) - P(0).\log_2 P(0) = 0$, that means the average information of this region is 0.

2.2 Effect of Gray coding on data bits in ideal cross point regions on the bit plane *n*−2

DEFINITION 2.2. *Let the positive integer N be the bit length of data points, the region of cross points $R_o\,(n)$, with n from 2 to $(N-1)$, is a set of data points whose grey values are from $(2^n - 2^{n-2})$ to $(2^n + 2^{n-2}-1)$. The point of grey value 2^n (if existing) is called the center point of the cross point region, and the grey value 2^n is called the central value. These regions are called the ideal cross point regions (ICRs) of type R..*

According to the definition 2.2, grey values of data points in ICRs are in the following set

$$V_R\,(n) = \{\,2^n - 2^{n-2},\ ...,\ 2^n + 2^{n-2}-1\}. \tag{6}$$

The exponent n of central values 2^n is from 2 to $(N-1)$. The rule $V_R\,(n)$ includes two groups: $V_{R\,l}\,(n)$ and ${}_{R\,g}\,(n)$ with $V_{R\,l}\,(n) = \{2^n - 2^{n-2}, 2^n - 2^{n-2}+1, ..., 2^n-2, 2^n-1\}$, $V_{R\,g}\,(n) = \{2^n, 2^n+1, ..., 2^n+2^{n-2}-2, 2^n+2^{n-2}-1\}$.

For example, when $n = 3$, $V_R\,(n) = \{2^3 - 2^{2-1}, ..., 2^3 + 2^{2-1}-1\} = \{6, 7, 8, 9\}$.

PROPOSITION 2.2. *Let n be the exponent of the central value 2^n in the ideal cross point region $R_o(n)$, n is from 2 to $(N-1)$, N is a positive integer of bit length. Bits of Gray codes in the ideal cross point region $R_o(n)$ on the bit plane $n-2$ are bits 0.*

Proof of Proposition 2.2.

Be based on Definition 2.2, with a value of n in the interval $2\div(N-1)$ grey values of data points in the region $R_o(n)$ are from $(2^n - 2^{n-2})$ to $(2^n + 2^{n-2}-1)$, so they may be expanded under the form of polynomials of radix 2 (7) and/or (8) as the followings

$$0.2^{N-1}+...+0.2^{n}+1.2^{n-1}+1.2^{n-2}+x.2^{n-3}+...+x.2^{1}+x.2^{0}, \tag{7}$$

$$0.2^{N-1}+...+0.2^{n+1}+1.2^{n}+0.2^{n-1}+0.2^{n-2}+x.2^{n-3}+...+x.2^{0}, \tag{8}$$

where x are bits 1 or 0. The expression (7) satisfies $V_{R\,l}\,(n)$ and the expression (8) satisfies $V_{R\,g}\,(n)$.

After Gray code transformation, (7) and (8) become

$$0.2^{N-1}+...+0.\,2^{n}+1.2^{n-1}+\mathbf{0.2^{n-2}}+x.2^{n-3}+...+x.2^{0}, \tag{9}$$

$$0.2^{N-1}+...+0.2^{n+1}+1.2^{n}+1.\,2^{n-1}+\mathbf{0.2^{n-2}}+...+x.\,2^{0}. \tag{10}$$

From (9) and (10) we can see both of them always give bits 0 in ICRs $R_o(n)$ on the bit plane $n-2$.

For example, when $n = 3$, $R_o(3) = \{6, 7, 8, 9\}$. Gray codes of the decimal values from 6 to 9 have bits 0 on the bit plane 1 (= 3 − 2). This is good to optimize the probability of data bits, because the probability of bit 1 in $R_o(n)$ is always 0, and the probability of bit 0 in $R_o(n)$ is always 1.

CONSEQUENCE 2.2. *In the ideal cross point regions Ro, after Gray code transformation the entropy of obtained data on a certain bit plane is minimum.*

Proof of Consequence 2.2.

This consequence is easily proved from the Definition 2.2 and Proposition 2.2. Before Gray coding, bit states around the central value 2^n in the region R_o on the bit plane $n-2$ are random. If viewing locally in the region of cross points $R_o(n)$, the probabilities of bit 1 and bit 0 are random, therefore the entropy of the corresponding bit string is often $H = -P(1).\log_2 P(1) - P(0).\log_2 P(0) > 0$.

After Gray code transformation, data bits in the region R_o on the bit plane $n-2$ are all bits 0, so the probability of bit 0 in this region is 1, and the probability of bit 1 in this region is 0, so the entropy of this bit string is always $H = -P(1).\log_2 P(1) - P(0).\log_2 P(0) = 0$, that means the average information of this region is 0.

3. CODING SYSTEM FOR LONG SOURCE SEQUENCE

Let us assume a source alphabet A, to be $A = \{a_1, a_2, ...\ a_c\}$, with c different symbols of a zero-memory information source, each symbol a_i is with a probability P_i and a quantity of appearance N_i. Let B be the code alphabet, B = {b_1, b_2, ..., b_d }, with d different symbols b_j, let α be the string of an arbitrary sequence of symbols that represent the concatenation of the string symbols. Each massage of source sequence $a_{i1}a_{i2}...a_{in}$ of length n is corresponding to a codeword $b_{j1}b_{j2}...b_{jl}$ which is a code sequence of length l. The notation a_i and b_j represent literal symbols from the arbitrary source and code alphabets for which the code is to be defined. The probabilities P_i are real numbers, the appearances N_i are integers.

Jones defined a cumulative frequency table F where F_i is computed by

$$Fi = \lfloor \frac{1}{2} + u \sum_{1 \le j \le i} P_j \rfloor, \quad 0 \le i \le c. \tag{11}$$

The function $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x, where x is a real number, F_0 = 0 and $F_c = u$, u is the scale factor that is mentioned below.

Here is the code generation procedure to exact arithmetic manipulation of integers for avoiding any inaccuracy which might be introduced into the algorithm by indiscriminate use of floating-point calculation. So at present, we have to accept an approximation for computing F_i by using probabilities P_i that are real numbers which are approximated. In this procedure the scale factor u that effectively converts a real probability to a frequency rate per u source symbols. This scale factor u can be chosen to avoid the rounding effect.

In fact, after the process of counting the quantity of appearance of source symbols we have a set N = {N_1, N_2, ... , N_c }, with $0 \le N_i \le u$, and with some simple operations we can see that the equation (11) becomes

$$Fi = \sum_{1 \le j \le i} N_j \ , \quad 0 \le i \le c. \tag{12}$$

So the procedure to compute F_i is not an approximate calculation and n is the number of source symbols in this case, but in some cases if we already had the probabilities P_i, we could use (11), not (12), to calculate F_i. And if we use (11), we have to choice the scale factor u appropriate to the values of the probabilities P_i .

The basic principle of source coding scheme is that both upper and lower limit of the interval corresponding to the string α are represented by three function $X(\alpha)$, $Y(\alpha)$ and $L(\alpha)$. $L(\alpha)$ is in the component of exponent $-L(\alpha)-w$ by which the base d is raised to give a scale factor. $X(\alpha)$ and $Y(\alpha)$ represent the position and width, respectively, of the interval, they are integers, and converted into fractions by the scale factor $d^{-L(\alpha)-w}$.

With the initial conditions

$$X(\in) = 0, \quad Y(\in) = d^W, \quad L(\in) = 0, \tag{13}$$

X, Y and L are defined recursively by the following equations (14), (15) and (16) :

$$X(\alpha a_i) = (X(\alpha) + \lfloor (Y(\alpha) F_{i-1}/u) + \frac{1}{2} \rfloor) d^s, \tag{14}$$

$$Y(\alpha a_i) = (\lfloor (Y(\alpha) F_i \ /u) + \frac{1}{2} \rfloor - \lfloor (Y(\alpha) F_{i-1}/u) + \frac{1}{2} \rfloor) d^s, \tag{15}$$

$$L(\alpha a_i) = L(\alpha) + s, \tag{16}$$

where αa_i is an extended sequence the interval limits of which are derived by a mapping of the new symbol a_i onto the previous interval based on the cumulative frequency table, s is the integer for which

$$d^w \le Y(\alpha a_i \ ) \le d^{W+1} \tag{17}$$

The parameter w is used to present the width of an interval by $Y(\alpha)$, it effectively determines the number of d-ary digits of precision because in the floating-point terms the scaling by d^s in (15) to satisfy (17) is equivalent to a normalization of $Y(\alpha a_i)$ to w+1 d-ary digits.

Besides that, Jones [9] presented main features, properties, encoding and decoding algorithms that we used to develop our scheme, but instead of using the probabilities to compute the cumulative frequencies we use the quantities of appearance of symbols, so we don't carry out approximate calculations for computing the cumulative frequencies F_i .

4. SCHEME LIC-ICR FOR LOSSLESS IMAGE COMPRESSION

Fig.1 presents the scheme LIC-ICR for image compression generally. Each step is numbered according to the sequence of the scheme, we have 6 steps from 1 to 6. The first step, Step 1 (Ideal cross point regions) looks for regions of cross point where we can optimize the probability of data bits. Grey values of data points in these regions satisfy (1) for ICR of type A (A_o), (6) for ICR of type R (R_o). Step 2 (Cross point map) will establish the map of ideal cross point regions. Here, we should notice that with every central value we have two types of ideal cross point region (A_o and R_o), they are used to encode data bits on the different bit planes (n−1 and n−2). This step evaluates and ignores small ideal cross point regions that contain 1 or 2 data points, these areas don't affect the processes of coding and decoding. The third step, Step 3: Gray coding, makes Gray code transformation [3]. Step 4 (Bit plane decomposition) decomposes image data into separate bit planes that are numbered from 0 to N−1, these numbers depend on the significance of bits, where N is the bit length of pixels of image. The next step, Step 5 : Optimizing probability, calculates the probability of data bits outside ICRs and optimizes probabilities of data bits in ICRs by the propositions 2.1 and 2.2. This optimization is based on the cross point map beforehand and implemented to Step 6 to compute frequencies of data bits. The last step, Step 6 : Coding, uses some algorithm in the process of entropy coding, like arithmetic coding. In the experimental results at the next section, we use Jones' algorithm [9] mentioned in the section 3 above. The process of encoding data bits should be carried out from the most to the least significant bit planes

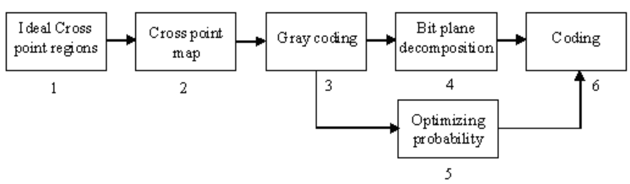

Fig.1 The scheme LIC− ICR with ideal cross point regions for lossless image compression.

because of the random of data on the less significant bit plane.

Therefore, this scheme is a process of entropy coding : modelling (from Step 1 to Step 5), and coding (Step 6).

5. EXPERIMENTAL RESULTS

Table 1 presents results when the images in Fig.2 are compressed by the scheme LIC-ICR. The compression ratio used here is the ratio between image files including headers of original image and image compressed. With these results we can see that the scheme LIC-ICR is good at images in which there are many same grey levels, especially medical images containing backgrounds with not much different grey levels.

The algorithm of the scheme LIC-ICR uses the fourth-order estimate of the source entropy [8], so there are 3 neighbor bits of the bit being coded, they are chosen beforehand. From the

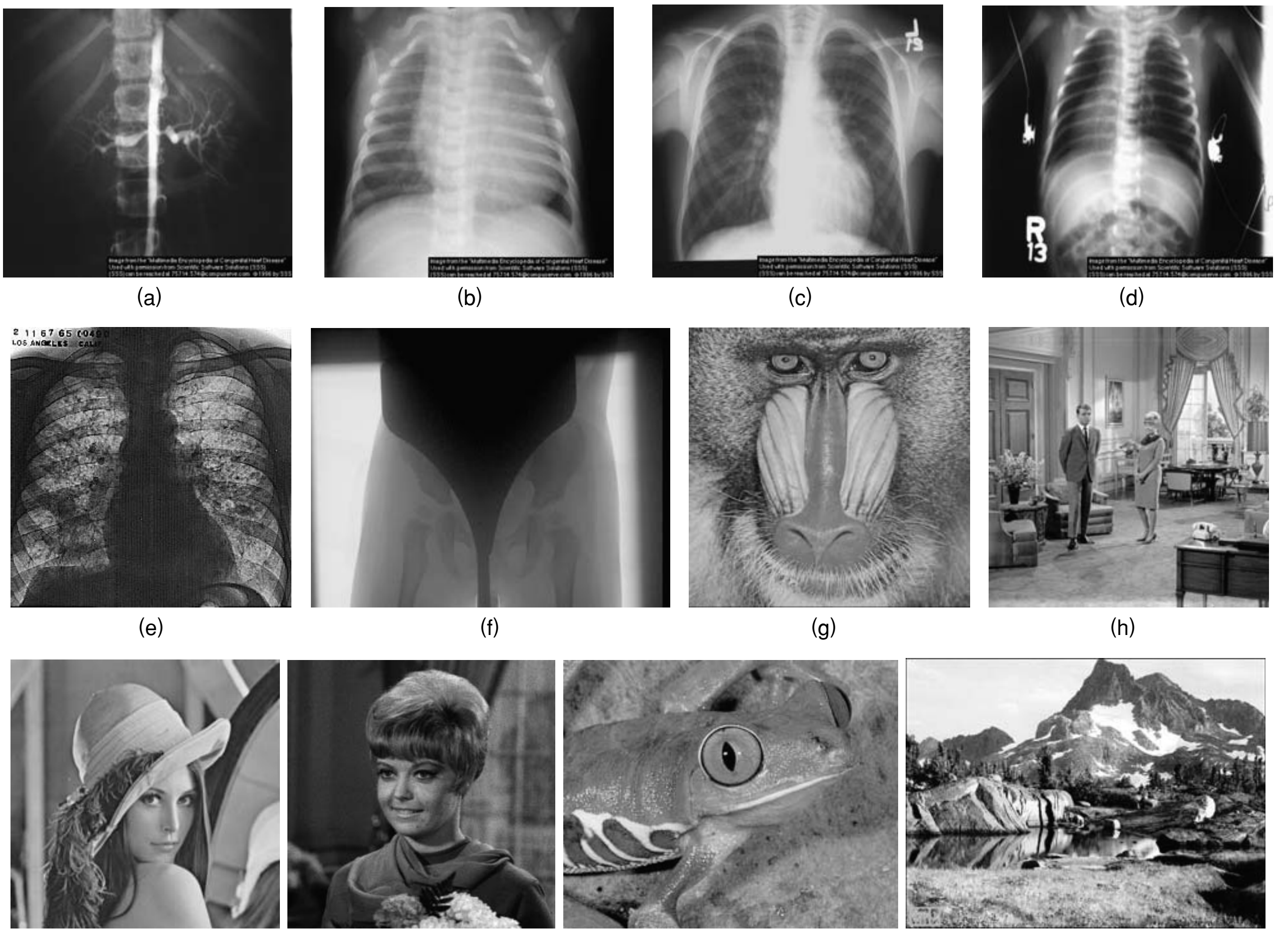
(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k) (l)

Fig.2 Images applied the scheme LIC–ICR with the optimization of probability of cross points
(a) Chest 1, (b) Chest 2, (c) Chest 3, (d) Chest 4, (e) Chest, (f) Joint, (g) Mandrill, (h) Couple, (i) Lena, (j) Zelda, (k) Frog, and (l) Mountain

TABLE 1 EXPERIMENTAL RESULTS OF LOSSLESS IMAGE COMPRESSION WITH SCHEME LIC–ICR

Image	Size	Compression ratio (LIC-ICR)	AES (WinRAR 3.40) [10]	JPEG 2000 [11]
CHEST 1	512 x 480	1.94390 : 1	1.67380 : 1	1.87753 : 1
CHEST 2	512 x 480	1.83422 : 1	1.45729 : 1	1.87491 : 1 *
CHEST 3	512 x 480	2.30261 : 1	2.20151 : 1	2.26839 : 1
CHEST 4	512 x 480	1.96219 : 1	1.72819 : 1	1.92259 : 1
CHEST	256 x 256	1.82454 : 1	1.57390 : 1	1.24317 : 1
JOINT	512 x 400	2.14345 : 1	1.93269 : 1	2.22525 : 1 *
MANDRILL	512 x 512	1.15822 : 1	1.20657 : 1 *	1.29101 : 1 *
COUPLE	512 x 512	1.62032 : 1	1.42748 : 1	1.51374 : 1
LENA	512 x 512	1.61084 : 1	1.56707 : 1	1.75220 : 1 *
ZELDA	256 x 256	1.84664 : 1	1.51162 : 1	1.60314 : 1
FROG	621 x 498	1.49413 : 1	2.12112 : 1 *	1.21746 : 1
MOUNTAIN	640 x 480	1.43333 : 1	1.52020 : 1 *	1.17767 : 1

* The results are better than the results of LIC-ICR

results in Table I we can see that if using the scheme LIC-ICR to losslessly compress medical images (Chest 1, Chest 2, Chest 3, Chest 4, Chest, Joint) we almost obtain higher compression ratios than another algorithms like AES (Advanced Encryption Standard) [10], used in the software WinRAR, and JPEG 2000, using lossless wavelet transform [11]. With photometric images (Mandrill, Couple, Lena, Zelda, Frog, Mountain), the scheme LIC-ICR may give better or worse results than another methods. This depends on the analyses of each method.

The encoding process here uses Jones' method [9] with some supplementaries in [12]. This algorithm is like the arithmetic coding but using integers in the processing. Until now, the program can try 8-bit grey images of bitmap file format.

The images restored must be identical with the original images. This problem is strictly carried out by comparing the original image with the image decompressed for every pixel because medical images are very important to diagnose.

6. CONCLUSION

The scheme LIC-ICR and some results are presented. This scheme with the definitions 2.1, and 2.2, the propositions 2.1, and 2.2, the consequences 2.1, and 2.2 is now a part of the theory of cross point regions which are mentioned in [13], and [14]. It illustrated the use of the theory of cross point regions with optimizing probabilities of data bits only in ideal cross point regions. Generally, the scheme LIC-ICR is a process of entropy coding, it includes two parts: modeling (Step 1-5) and coding (Step 6). The cross point region theory can be used in the first part in order to reduce interpixel redundancy, the second part uses some algorithm, like arithmetic coding or Jones' method to reduce coding redundancy.

The basic concepts have been introduced in [1], [2] which are now the base for the theory of optimizing probabilities of cross points with the new concept of ideal cross point regions. A meaningful improvement in compression ratio has been obtained, compared to the other authors' methods. From these concrete bases, the problem of improving the compression ratio of image processing and transmission can furtherly be developed in the future.

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