Investigation of basic imaging properties in digital radiography. 6. MTFs of II–TV digital imaging systems[a),b)]

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We devised a new, simple technique for measuring the modulation transfer function (MTF) of a digital imaging system by using an image of an angulated slit. With this technique, the "presampling" analog MTF, which includes the geometric unsharpness, the detector unsharpness, and the unsharpness of the sampling aperture, can be measured even beyond the Nyquist frequency. A single-frame image of a slightly angulated slit was employed in order to obtain Fourier transforms of line spread functions at different alignments. The presampling MTF was determined by averaging the two Fourier transforms which we obtained from two extreme alignments (center and shifted) of the slit relative to the sampling coordinate. The presampling MTFs of our digital subtraction angiographic system were determined in two orthogonal directions for three different image-intensifier modes.

Key words: modulation transfer function, resolution, digital radiography, II–TV system

I. INTRODUCTION

The modulation transfer function (MTF) has been used as a powerful tool for the characterization of the resolution properties of imaging systems and their components, such as screen–film systems.[1–3] However, it was found from our previous investigations[4] that, in digital radiographic imaging systems, the interpretation of the MTF requires caution because of the discrete data sampling, which causes an aliasing effect. Recently, Sones and Barnes presented a method of measuring the "presampling" analog MTF of a Picker digital chest imaging system up to and, for undersampled systems, beyond the Nyquist frequency.[5,6] In their measurements, they employed a test object with multiple wires which was constructed carefully by taking into account parameters such as the sampling distance and the geometry. The complexity of the selection of the many parameters may be considered a problem associated with their method.

We have developed a new method of determining the presampling analog MTF of digital imaging systems to frequencies beyond the Nyquist frequency. Our method can be implemented easily using a single-slit device to determine the presampling analog MTF component in digital imaging systems. Below, we describe our method and present experimental results obtained for a digital subtraction angiographic (DSA) system.

II. THEORY

From transfer function analysis, it is known that the resolution properties of a linear, shift invariant imaging system can be determined from the output of an input delta function.[1] The Fourier transform of the normalized output, i.e., of the point spread function (PSF) in a two-dimensional case or the line spread function (LSF) in a one-dimensional case, is called the optical transfer function (OTF). The OTF has two components, the modulation transfer function (MTF) and the phase transfer function (PTF), which represent the modulus and the phase, respectively, of the OTF. Since the resolution properties are determined from a slit image, we will discuss the one-dimensional case and thus consider the LSF.

In an analog imaging system, the OTF can be determined from the Fourier transform of an LSF obtained from a narrow slit image:

$$\mathrm{OTF}(u) = \int_{-\infty}^{\infty} \mathrm{LSF}(x) \exp(-j2\pi ux)\, dx, \qquad (1)$$

where

$$\int_{-\infty}^{\infty} \mathrm{LSF}(x) dx = 1.$$

With a radiographic imaging system, the spatial distribution of the slit image is degraded by the geometric unsharpness[7] (due to the focal-spot size and the magnification factor) and the detector response. The product of the OTF of geometric unsharpness and the OTF of the detector system is the analog OTF prior to sampling and digitization. In a digital imaging system, the slit image is further degraded by the response of the sampling aperture; the resulting OTF, OTF(u), which is obtained from the LSF of the blurred slit image, will be referred to here as the presampling analog OTF.[4,8] In addition, the slit image is sampled at discrete intervals of the sampling distance, Δx; the corresponding OTF will be called the "digital" OTF.[4,8] The sampling process can be thought of as a multiplication of the spatial distribution of the LSF of a blurred slit image with the sampling function or comb function[9,10] represented by $\Sigma\delta(x - k\Delta x)$, in which k is an integer. Thus, this sampled LSF is given by LSF($k\Delta x$). A sampled LSF is shown schematically by the discrete points in Fig. 1(a); here the sampling occurred at the "center" alignment, i.e., the center of the presampled LSF coincided with the center of the sampling aperture.[8] We assume that a sufficient number of quantization levels, such as the ten bits used in our experiments, are available so that the

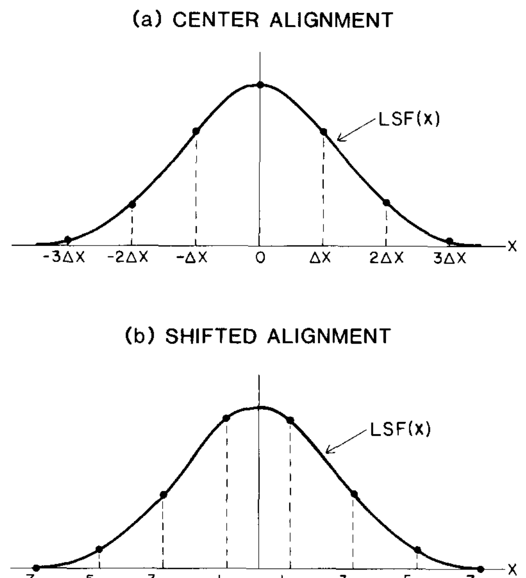

FIG. 1. An analog LSF (solid curve) sampled at increments of the sampling distance Δx, where (a) the center of the sampling pixel coincides with the center of the LSF and (b) the center of the sampling pixel is shifted laterally from the center of the LSF by one-half the sampling distance.

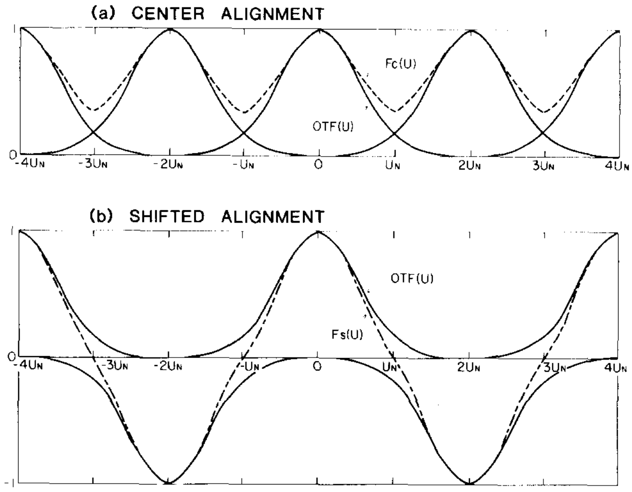

FIG. 2. Fourier transforms $F_c(u)$ and $F_s(u)$, which demonstrate the aliasing effect, are shown, respectively, for (a) center and (b) shifted alignments of the LSF relative to the sampling coordinate. Aliases of the presampling analog OTF for the center alignment are centered at $2k$ $(k \neq 0)$ times the Nyquist frequency (u_N), whereas aliases for the shifted alignment appear alternately as positive and negative values at $2k$ $(k \neq 0)$ times the Nyquist frequency.

effect of quantization on the OTF will be negligible. The discrete Fourier transform of the sampled LSF, $F_c(u)$, can be expressed as

$$\begin{aligned} F_c(u) &= \sum_{k=-\infty}^{\infty} \mathrm{LSF}(k\Delta x)\exp(-j2\pi uk\Delta x)\Delta x \\ &= \int_{-\infty}^{\infty} \mathrm{LSF}(x)\left\{\sum_{k=-\infty}^{\infty} \delta(x-k\Delta x)\Delta x\right\} \\ &\quad \times \exp(-j2\pi ux)dx \\ &= \int_{-\infty}^{\infty} \mathrm{OTF}(u-u')\left\{\sum_{k=-\infty}^{\infty} \delta(u'-k/\Delta x)\right\}du' \\ &= \sum_{k=-\infty}^{\infty} \mathrm{OTF}(u-2ku_N), \end{aligned} \tag{2}$$

where the Nyquist frequency u_N is defined as the reciprocal of twice the sampling distance ($u_N = 1/2\Delta x$). Here the convolution–multiplication theorem[9,10] of Fourier analysis has been employed along with the Fourier transform of the sampling function given by $(1/\Delta x)\Sigma\delta(u - k/\Delta x)$, where k is an integer. The formation of $F_c(u)$ is demonstrated in Fig. 2(a), where the solid curves represent aliases of the presampling OTF and the dashed curve indicates $F_c(u)$, which corresponds to the digital OTF. Aliases of the presampling OTF are centered at $2k$ $(k \neq 0)$ times the Nyquist frequency. If the tails of the presampling OTF are not zero beyond the Nyquist frequency, the overlapped portions of the aliases will sum together, resulting in a foldover of high-frequency components to low frequencies; this is the aliasing effect.[10]

A digital imaging system is not shift invariant, however, and so the slit may be positioned such that the center of the slit does not coincide with the center of the sampling aperture. An extreme case of alignment exists if the center of the slit is shifted by one-half the sampling distance from the center of the sampling aperture.[8] For this "shifted" alignment, one can describe the sampled LSF by using a shifted set of sampling coordinate, i.e., $\mathrm{LSF}([k+1/2]\Delta x)$, as is shown schematically in Fig. 1(b). By applying the discrete Fourier transformation and the shift theorem[9,10] to the sampled LSF at the shifted alignment, one obtains

$$\begin{aligned} F_s(u) &= \sum_{k=-\infty}^{\infty} \mathrm{LSF}([k+1/2]\Delta x)\exp(-j2\pi uk\Delta x)\Delta x \\ &= \int_{-\infty}^{\infty} \mathrm{LSF}(x)\left\{\sum_{k=-\infty}^{\infty} \delta(x-[k+1/2]\Delta x)\Delta x\right\} \\ &\quad \times \exp(-j2\pi ux)dx \\ &= \int_{-\infty}^{\infty} \mathrm{OTF}(u-u')\left\{\exp(-j\pi u'\Delta x) \right. \\ &\quad \left. \times \sum_{k=-\infty}^{\infty} \delta(u'-k/\Delta x)\right\}du' \\ &= \sum_{k=-\infty}^{\infty} (-1)^k \mathrm{OTF}(u-2ku_N), \end{aligned} \tag{3}$$

where k is an integer. Figure 2(b) illustrates the formation of the Fourier transform of the sampled LSF, $F_s(u)$, for this shifted alignment. It should be noted that the positive and negative aliases of the presampling OTF occur alternately at $2k$ $(k \neq 0)$ times the Nyquist frequency. Thus, $F_s(u)$ is zero at $2k+1$ times the Nyquist frequency.

For the calculation of the Fourier transform of the LSF at this extreme alignment, we have shifted the sampling coordinate as illustrated previously in Fig. 1(b). However, if the slit image (LSF) instead of the sampling coordinate had been shifted, the corresponding Fourier transform of the LSF would include an additional phase factor, and thus the following method of analysis would not be applicable.

By averaging the two Fourier transforms in Eqs. (2) and (3) (i.e., the two extreme cases), we obtain $F_M(u)$, namely,

$$F_M(u) = \tfrac{1}{2}[F_c(u) + F_s(u)]$$
$$= \sum_{m=-\infty}^{\infty} \mathrm{OTF}(u - 4mu_N)$$
$$= \mathrm{OTF}(u) + \mathrm{OTF}(u - 4u_N) + \mathrm{OTF}(u + 4u_N)$$
$$+ \mathrm{OTF}(u - 8u_N) + \mathrm{OTF}(u + 8u_N) + \cdots. \quad (4)$$

If OTF $(u) = 0$ for $|u| \geqslant 2u_N$, we obtain

$$F_M(u) = \tfrac{1}{2}[F_c(u) + F_s(u)]$$
$$= \mathrm{OTF}(u), \quad (5)$$

within the frequency range of $\pm 2u_N$. Equation (5) indicates that the presampling OTF can be determined by averaging of the two Fourier transforms which are obtained from slit images measured with the center and shifted alignments; this is shown in Fig. 3. The presampling MTF can then be calculated from the absolute value of $F_M(u)$. It should be noted that this method is valid when the presampling OTF is negligibly small at frequencies greater than twice the Nyquist frequency. However, if the presampling OTF does have a small component beyond $2u_N$, then the terms OTF$(u - 4u_N)$ and OTF$(u + 4u_N)$ will contribute to the summation in Eq. (5) as small errors attributable to the aliasing effect. If the presampling OTF has a substantial amount of components at frequencies higher than twice the Nyquist frequency, an expansion of this method to higher frequencies is necessary, as will be described in Sec. V.

III. EXPERIMENTAL METHOD

In this study, we employed a Siemens Digitron 2 DSA system as an image intensifier (II)–TV digital imaging system. This system consists of a Garantix 1000 x-ray generator with a 0.6-mm focal spot x-ray tube, a triple-mode (25, 17, and 12 cm) Optilux RBV 25/17 HN II, and a Videomed N TV system. No grid was used for the MTF measurements. The Digitron 2 includes a logarithmic amplifier and a 10-bit analog-to-digital (A/D) converter. All images were acquired at 63 kV in the pulsed x-ray exposure mode with an image matrix size of 512×512 or 256×256.

The pixel size in an II–TV digital imaging system is directly proportional to the II field size used and inversely related to the number of pixel elements in the image matrix. For a given II mode, the effective pixel size at the II input phosphor plane in the two orthogonal directions, i.e., perpendicular and parallel to the TV raster lines, was measured by exposure of a periodic line pattern composed of aluminum and lead.

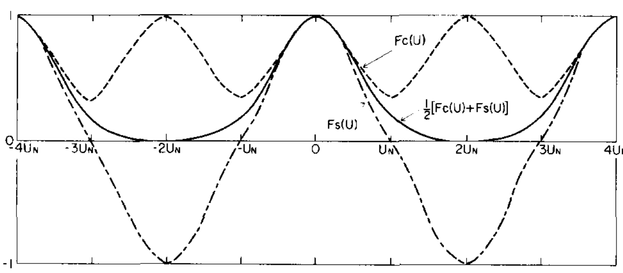

FIG. 3. Illustration of the method for determining the presampling OTF by averaging the two Fourier transforms, $F_c(u)$ and $F_s(u)$, of LSFs obtained with the center and shifted alignments, respectively. The presampling OTF was assumed to be negligible at frequencies greater than twice the Nyquist frequency ($2u_N$).

The method which we developed for measurement of the presampling MTF in II–TV digital imaging systems is shown schematically in Fig. 4. A long, narrow slit (0.05 mm wide, 100 mm long) was made from two lead plates (100 mm×35 mm×2 mm) which were glued to an aluminum support. The slit was placed centrally in front of the II input surface. The width of the slit was large enough to allow transmission of x-ray photons, but sufficiently small to avoid erroneous degradation of the slit image. Without precision instruments, it is difficult to position the slit exactly for either the center or the shifted alignment. However, we found that a slightly angulated slit configuration can be achieved easily to provide a series of LSFs at various alignments. Thus, the slit was placed at a slight angle (about 1°–3° for the 12-cm II mode with the 512×512 matrix size) to the particular direction of interest (i.e., perpendicular or parallel to the TV raster lines).

We acquired the slit images with the x-ray beam collimated to a small area around the slit opening, in an effort to avoid the contribution of veiling glare[11] from the outside of the slit device. Since our DSA unit includes an automatic exposure control, it was necessary to "fool" this control by initially exposing with a broad beam and a 0.81-mm-thick copper sheet. After the iris adjustment had been completed by the automatic exposure control during the initial few frames, the exposure field was collimated down to about the size of the slit opening, and the copper sheet was removed from the field.

A characteristic curve,[12] which relates the output pixel value to the input relative x-ray intensity, was employed as a means of linearization. For determination of the characteristic curve of the II–TV digital system, an aluminum step wedge having nine steps with thickness increments of 6.3 mm was exposed at 63 kV. The experimental results and the method of measurement for the characteristic curve in II–TV digital systems are described in detail elsewhere.[12]

Digital slit images obtained with the Digitron 2 were transferred to a Siemens Evaluskop computer system which facilitated the reading of the pixel values. Slit image data were analyzed along a direction perpendicular to the length of the slit, so that we could determine a series of LSFs at the various alignments. The coordinates for the two extreme LSF alignments were set as shown in Fig. 1 prior to Fourier transformation. The LSF (or slit image) showing a sharp sin-

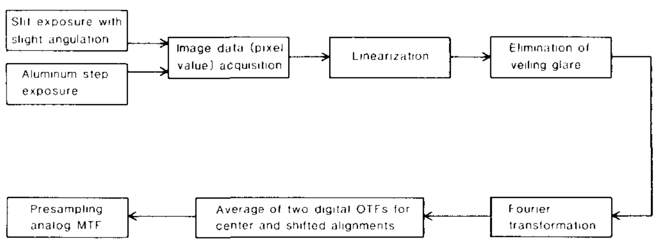

FIG. 4. Schematic diagram illustrating the measurement of the presampling MTF in II–TV digital systems for the case in which the MTF is negligibly small at frequencies larger than twice the Nyquist frequency.

gle peak corresponds to the center alignment, and the LSF with a broad peak containing two equal values corresponds to the shifted alignment. We found that the digital LSF was slightly asymmetric, expecially when the slit was positioned perpendicular to the TV raster lines, although the effect of this asymmetry on the phase of the OTF was very small. The slit image data in terms of pixel values were converted to those in terms of relative x-ray intensities by using the characteristic curve. Small veiling-glare contributions and an overall background trend were eliminated from the LSF data by using a linear curve-fitting technique.

After we had performed the above procedures, the Fourier transforms of each corresponding LSF data were calculated. The resulting highest and lowest Fourier transforms (or digital OTFs) correspond to the center and shifted alignments, respectively. These two Fourier transforms were averaged [see Eq. (5)] to yield the presampling OTF up to twice the Nyquist frequency.

The presampling OTF or MTF which we determined includes the unsharpness in the II and optical components, the unsharpness in the TV camera and electronic components, and the blurring effect of the sampling aperture; it does not include the effect of unsharpness in the display system, which will be discussed later. Because the distance between the focal spot and the II input surface was kept at 90 cm, the effect of the geometric unsharpness created by the size and shape of the focal spot was assumed to be negligible compared to the other factors contributing to unsharpness.

IV. EXPERIMENTAL RESULTS

We found that the effective pixel size at the II input surface was approximately 7% larger in the horizontal direction (parallel to the TV raster lines) than in the vertical direction for all II modes. For our MTF measurements, we used the average of the two values for simplicity. The effective pixel sizes measured with a 512×512 matrix size were 0.32, 0.38, and 0.59 mm for the 12-, 17-, and 25-cm II modes, respectively. For the 256×256 images, we used pixel sizes which were twice those of the corresponding 512×512 images.

Figure 5 shows a slit image obtained with a multiformat camera. The slit was positioned in a direction approximately parallel to the TV raster lines and exposed in the 12-cm II mode with the 512×512 matrix size. As mentioned earlier, because of the slight angulation of the slit, a range of LSFs corresponding to various alignments of the slit relative to the sampling coordinates was obtained from the single slit image illustrated in Fig. 5. Figure 6 shows the range of Fourier transforms determined from this slit image. The highest and lowest curves correspond to the digital MTFs for the center and shifted alignments, respectively. All of the digital MTFs were normalized by the area of the corresponding LSF. The presampling MTF obtained with the data of Figs. 5 and 6 is shown in Fig. 7. The figure indicates that the presampling MTF can be measured even beyond the Nyquist frequency (1.56 cycles/mm), and that the MTF near twice the Nyquist frequency is negligibly small.

Presampling MTFs of our DSA system are shown in Fig. 8 for the three different II modes and for the two slit orientations used. Each curve is an average of five MTFs calculated from five independent placements of the slit at the center of the II. The average standard deviation for the modulation transfer factor at spatial frequencies less than the Nyquist frequency was approximately 0.015. Again, it should be noted that the MTF was measured beyond the Nyquist frequency for all six cases. It is apparent from Fig. 8 that, the smaller the II mode, the greater the MTF. In addition, for a given II mode, the MTF measured with the slit in the direction parallel to the TV raster lines is greater than that obtained in the perpendicular direction.

V. DISCUSSION

In our determination of the presampling MTF, we eliminated the component of veiling glare[11] from the LSF, and thus the measured MTF does not include the corresponding low-frequency component. With an II–TV system, the veiling glare, which is mainly attributable to scattering of x rays, electrons, and light in the II, degrades the image contrast. The low-frequency component due to the veiling glare is sometimes included in the MTF of the II.[13] However, we believe that this component should be measured and analyzed separately from the MTF of the II–TV digital system, since its contribution can be obtained accurately with inde-

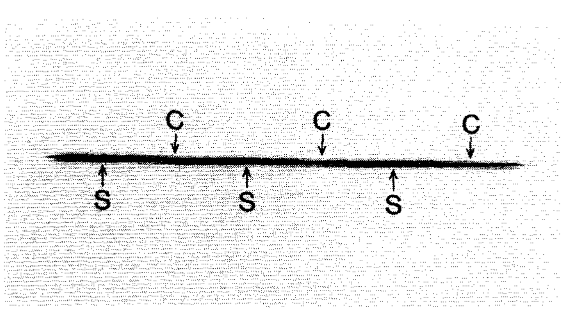

FIG. 5. A slit image acquired with the 12-cm II mode and a 512×512 matrix size. The slit was angulated slightly (1.3°) from the direction parallel to the TV raster lines and was used to determine the MTF in a direction perpendicular to the raster lines. C and S indicate the positions of the center and shifted alignments, respectively.

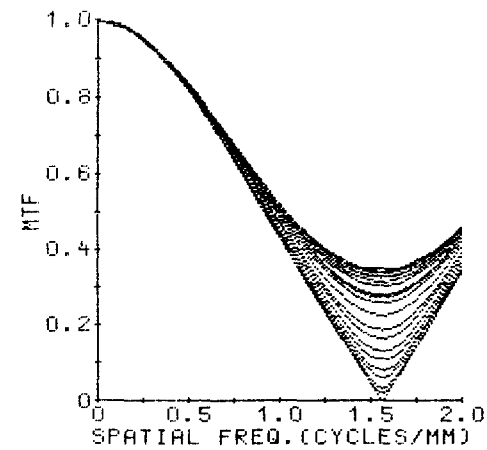

FIG. 6. Range of digital MTFs obtained with different alignments of the slit image shown in Fig. 5. The highest and lowest curves correspond to digital MTFs for the center and shifted alignments, respectively. The Nyquist frequency is 1.56 cycles/mm.

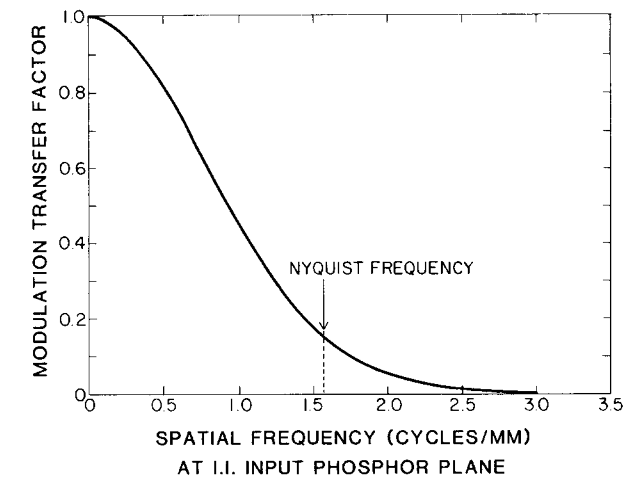

FIG. 7. Measured presampling MTF of our II–TV digital system with a 512×512 matrix size and a 12-cm II mode. Note that the MTF is determined even beyond the Nyquist frequency (1.56 cycles/mm).

pendent measurements. The effect of veiling glare on image contrast degradation can be evaluated in a way similar to that of off-focus radiation[14] and scattered radiation.[15–17]

In principle, if the presampling OTF cannot be regarded as zero at frequencies higher than twice the Nyquist frequency, the method we have described is not applicable. However, if one considers the LSFs which are obtained when the coordinate system is shifted by plus and minus one-fourth the pixel size, as well as the LSFs for the center and shifted (one-half the pixel size) cases, then the limitation on the presampling MTF can be extended to being zero only beyond *four times* the Nyquist frequency (see the Appendix). In this case, the presampling OTF can be obtained from averaging of the four corresponding Fourier transforms. In general, the presampling OTF needs only be zero beyond 2^n times the Nyquist frequency, if 2^n sampled LSFs are obtained corresponding to 2^n different shifts of the sampling coordinate at increments of $\Delta x/2^n$ from the center of the LSF. A general expression for determining the presampling OTF is also shown in the Appendix. However, an accurate determination of finer shifted alignments may not be performed so easily.

We applied the method which includes the quarter shifts to the slit image obtained with the 256×256 matrix size, since the presampling MTF at frequencies larger than twice the Nyquist frequency may not be negligible for this matrix size. Figure 9 shows a range of digital MTFs determined from the slit image which was almost parallel to the TV raster lines and which was obtained with a 256×256 matrix size and a 17-cm II mode. For the center alignment, only one data point represented the LSF, because the distribution of the unsharpness in the LSF was less than the pixel size; i.e., the sampling aperture MTF was dominant. Thus, the corresponding digital MTF is unity because of the large amount of aliases.[4,8] For the shifted alignment (one-half the pixel size), the LSF consisted of only two points, and thus the digital MTF is equal to $|\cos(\pi\Delta x u)|$,[8] which indicates that the MTF of the rectangular sampling aperture is dominant. The corresponding presampling MTF, obtained up to a spatial frequency of four times the Nyquist frequency, is shown in Fig. 10.

If the presampling MTF can be regarded as zero at spatial frequencies above twice the Nyquist frequency, the area under the sampled LSF for the center alignment would be expected to be equal to that for the shifted alignment; i.e., $F_c(0) = F_s(0)$. However, our experimental results indicated that the Fourier transforms of the sampled LSFs contained different values at zero spatial frequency when the LSFs were not normalized individually. This difference may be due to the nonuniform background in the II–TV system and also to the discrete summation of the sampled LSF data. Therefore, we normalized the MTFs (or LSFs) obtained with various alignments, as described earlier, and we confirmed that there was no significant difference in the calculated presampling MTFs as a result of this normalization procedure.

In the following, we shall demonstrate that an "overall" MTF in a digital imaging system may incorrectly indicate the resolution properties of the system if the aliasing effect is not negligible, and thus that the overall MTF cannot be used

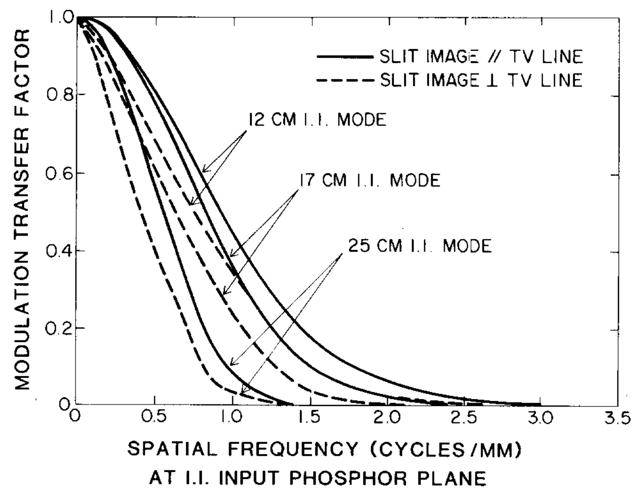

FIG. 8. Presampling MTFs of our DSA system for two orthogonal slit directions (parallel and perpendicular to the TV raster lines), for three different II modes and a 512×512 matrix size.

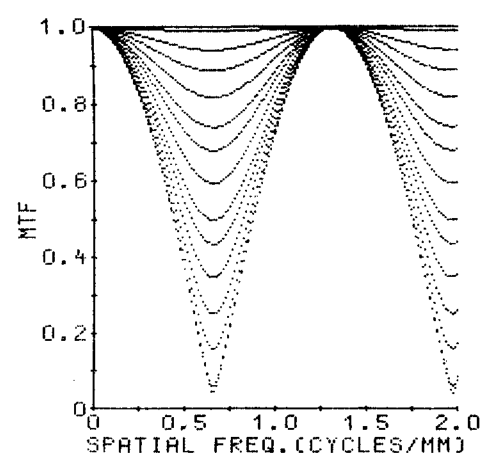

FIG. 9. Range of digital MTFs corresponding to different alignments of the slit for a 17-cm II mode and a 256×256 matrix size. The slit was placed in a direction nearly parallel to the TV raster lines. The Nyquist frequency is 0.66 cycle/mm.

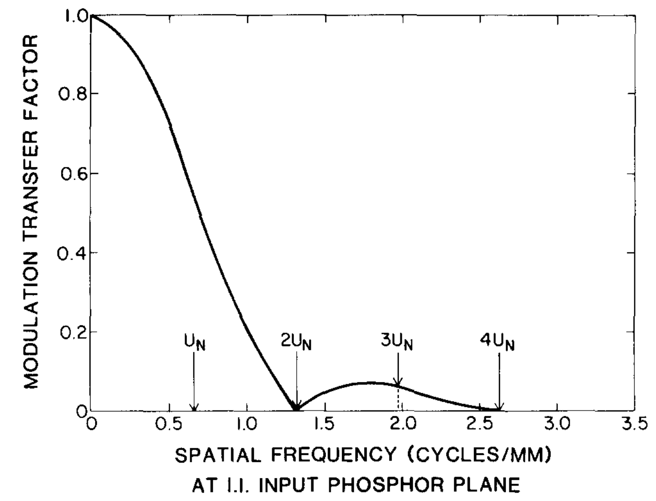

FIG. 10. Measured presampling MTF of our II–TV digital system for a 256×256 matrix size and a 17-cm II mode. Note that the MTF is determined up to four times the Nyquist frequency ($4u_N$).

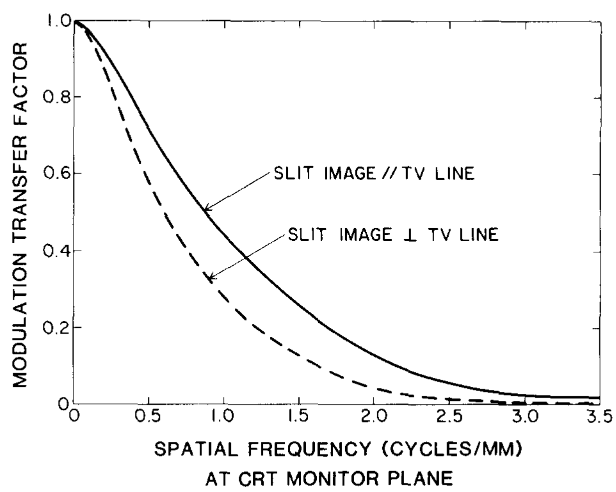

FIG. 11. MTFs of our CRT monitor (256×256 matrix size and 0.5-mm pixel size), measured in two orthogonal directions.

in the same manner as that in an analog imaging system. The overall MTF of a digital system can usually be determined from the presampling MTF if one takes into account the discrete sampling, any filtering operations that are applied, and the size and shape of the display aperture.[4] However, our method directly provides the digital MTF from which the presampling MTF is calculated. Thus, the overall MTF will be given by the product of the digital MTF and the MTF of the display device. We measured the MTF of the CRT monitor (256×256 matrix size) installed in our Evaluskop computer system by displaying a "slit" image of constant digital values and a width of one pixel on the CRT monitor. The pixel size of the CRT monitor was approximately 0.5 mm. A film image of the slit was obtained with Kodak RPC film and a photographic camera. The slit image was scanned with a microdensitometer in a direction perpendicular to the slit, yielding a slit trace in terms of film density. After linearization of the slit trace by use of a characteristic curve, which related the density of the film to the relative brightness on the display device, we determined the display MTF from the Fourier transform of the slit image. The MTFs of our display device, measured in the two orthogonal directions, are plotted in Fig. 11 on a spatial-frequency axis in the display plane. The MTF of the display device in the direction parallel to the TV raster lines is greater than that in the direction perpendicular to the raster lines, similar to the trend shown in Fig. 8 for the presampling MTFs. The overall MTFs for two different alignments for a 512×512 matrix size and a 12-cm II mode are depicted in Fig. 12, where the solid and dashed curves represent the center and shifted alignments, respectively. It appears as if, at high spatial frequencies, the overall MTF in Fig. 12 is greater than the corresponding presampling MTF shown in Fig. 7. However, this increase in the overall MTF is due to aliasing artifacts and does not represent an actual increase in the resolution properties. We have previously discussed in detail that, although the presampling MTF can characterize inherent resolution properties of the analog component of a digital imaging system, the overall MTF in a digital radiographic system may incorrectly indicate the resolution capability.[4] However, if the overall MTF of a digital imaging system contains a negligible amount of aliasing artifacts, it can be used together with a specific input signal to determine the specific output in the same way that the MTFs of analog systems are used.

The basic concept used in our method for increasing the limiting Nyquist frequency in the determination of the presampling OTF can be applied also in improving the spatial resolution of digital images. For example, the resolution of II–TV digital systems can be improved by a "double reading" of the II image with a TV camera such that the second reading (or scan) is made at positions shifted by one-half the pixel size from the first scan. This scanning method is equi-valent to sampling at both the center and shifted alignments in order to increase the Nyquist frequency by a factor of 2. This type of double reading may be implemented by shifting mechanically or optically the positions of a TV camera relative to the II, or by employing a double-reading (or high resolution) TV camera. Another potential approach is to obtain two image frames between which the position of the II–TV system is shifted relative to the patient. In fact, in computed tomography, investigators have implemented this

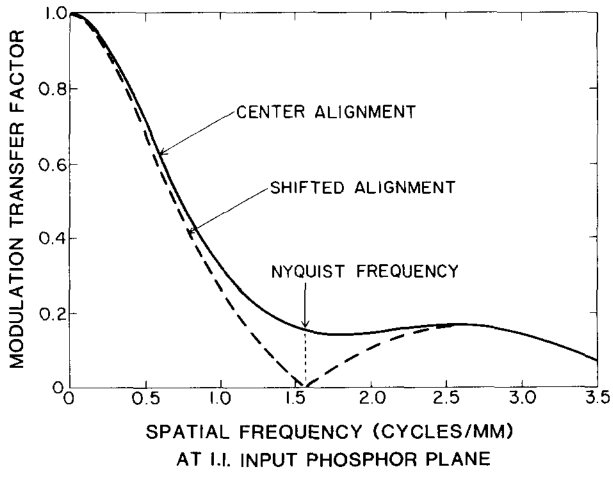

FIG. 12. Overall MTFs of our II–TV digital system for two different alignments, for a 512×512 matrix size and a 12-cm II mode.

concept of double sampling by offsetting the detector sampling points and interleaving the opposing views of a 360° scanner.[18]

VI. CONCLUSION

We developed a new, simple method for determining the presampling MTF in II–TV digital systems up to spatial frequencies even beyond the Nyquist frequency. The method may be applied to digital imaging systems which have a presampling analog MTF whose frequency components extend beyond the Nyquist frequency. We believe that knowledge of the presampling analog MTF, together along with the corresponding pixel size, will be useful in the determination of the signal-to-noise ratio (SNR), the evaluation of different digital systems, and the design of future radiographic imaging systems.

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APPENDIX

We will show that the presampling MTF whose components are zero at spatial frequencies larger than four times the Nyquist frequency can be obtained from averaging of four Fourier transforms of LSFs corresponding to the shifting of the sampling coordinates from the center of a slit image by zero, $+\Delta x/2$, $+\Delta x/4$, and $-\Delta x/4$. Fourier transforms for center and shifted alignments have already been presented in Eqs. (2) and (3). Fourier transforms for the shifted alignments of $\pm\Delta x/4$, $F_s(u)_{+\Delta x/4}$, and $F_s(u)_{-\Delta x/4}$, are expressed by

$$F_s(u)_{+\Delta x/4} = \int_{-\infty}^{\infty} \mathrm{OTF}(u-u') \left\{ \exp(-j\pi u'\Delta x/2) \times \sum_{k=-\infty}^{\infty} \delta(u'-k/\Delta x) \right\} du' = \sum_{k=-\infty}^{\infty} \exp(-j\pi k/2)\mathrm{OTF}(u-k/\Delta x) \quad \text{(A1)}$$

and

$$F_s(u)_{-\Delta x/4} = \sum_{k=-\infty}^{\infty} \exp(j\pi k/2)\mathrm{OTF}(u-k/\Delta x), \quad \text{(A2)}$$

respectively. By averaging the four Fourier transforms given by Eqs. (2), (3), (A1), and (A2), one obtains $F_M(u)$,

$$\begin{aligned} F_M(u) &= \tfrac{1}{4}[F_c(u)+F_s(u)+F_s(u)_{+\Delta x/4}+F_s(u)_{-\Delta x/4}] \\ &= \tfrac{1}{4}\sum_{k=-\infty}^{\infty} \mathrm{OTF}(u-k/\Delta x) \times [1+(-1)^k+2\cos(\pi k/2)] \\ &= \sum_{m=-\infty}^{\infty} \mathrm{OTF}(u-8mu_N) \\ &= \mathrm{OTF}(u)+\mathrm{OTF}(u-8u_N)+\mathrm{OTF}(u+8u_N) \\ &\quad +\mathrm{OTF}(u-16u_N)+\mathrm{OTF}(u+16u_N)+\cdots . \end{aligned} \quad \text{(A3)}$$

If $\mathrm{OTF}(u)=0$ for $|u|\geqslant 4u_N$, we obtain

$$F_M(u) = \tfrac{1}{4}[F_c(u)+F_s(u)+F_s(u)_{+\Delta x/4}+F_s(u)_{-\Delta x/4}] = \mathrm{OTF}(u) \quad \text{(A4)}$$

within the frequency range of $\pm 4u_N$.

In general, when the origin of the sampling coordinates is shifted by $(2p+1)\Delta x/2^n$ from the center of the slit image (p = integer), the Fourier transform of the LSF is given by

$$\begin{aligned} F_s(u)_{(2p+1)\Delta x/2^n} &= \int_{-\infty}^{\infty} \mathrm{OTF}(u-u') \times \left\{ \exp[-j\pi u'(2p+1)\Delta x/2^{n-1}] \times \sum_{k=-\infty}^{\infty} \delta(u'-k/\Delta x) \right\} du' \\ &= \sum_{k=-\infty}^{\infty} \exp[-j\pi k(2p+1)/2^{n-1}] \times \mathrm{OTF}(u-k/\Delta x), \end{aligned} \quad \text{(A5)}$$

where $p=0$ for $n=1$, and $-2^{n-2}\leqslant p\leqslant 2^{n-2}-1$ for positive integers n greater than one. $F_M(u)$ can be obtained by averaging of the 2^n Fourier transforms (n = positive integer); we then have

$$\begin{aligned} F_M(u) &= (1/2^n)[F_c(u)+F_s(u)+F_s(u)_{+\Delta x/4}+F_s(u)_{-\Delta x/4} + \cdots + F_s(u)_{(2p+1)\Delta x/2^n}] \\ &= (1/2^n)\sum_{k=-\infty}^{\infty} \mathrm{OTF}(u-k/\Delta x) \times [1+(-1)^k+2\cos(\pi k/2) \\ &\quad + 2^2\cos(\pi k/2)\cos(\pi k/2^2)+\cdots \\ &\quad + 2^{n-1}\cos(\pi k/2)\cos(\pi k/2^2)\cdots\cos(\pi k/2^{n-1})] \\ &= \sum_{m=-\infty}^{\infty} \mathrm{OTF}(u-m\cdot 2^{n+1}u_N) \\ &= \mathrm{OTF}(u)+\mathrm{OTF}(u-2^{n+1}u_N) \\ &\quad +\mathrm{OTF}(u+2^{n+1}u_N)+\mathrm{OTF}(u-2\cdot 2^{n+1}u_N) \\ &\quad +\mathrm{OTF}(u+2\cdot 2^{n+1}u_N)+\mathrm{OTF}(u-3\cdot 2^{n+1}u_N) \\ &\quad +\mathrm{OTF}(u+3\cdot 2^{n+1}u_N)+\cdots . \end{aligned} \quad \text{(A6)}$$

If $\mathrm{OTF}(u)=0$ for $|u|\geqslant 2^n u_N$, we obtain

$$F_M(u) = \mathrm{OTF}(u), \quad \text{(A7)}$$

within the frequency range of $\pm 2^n$ times the Nyquist frequency. This implies that the presampling MTF can be determined by averaging of the 2^n Fourier transforms of the LSFs at corresponding different alignments, provided it is negligible beyond 2^n times the Nyquist frequency.

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