Reprinted from
JAPANESE JOURNAL OF APPLIED PHYSICS
VOL. 19, NO. 7, JULY, 1980 pp. 1403–1410

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Japanese Journal of Applied Physics
Vol. 19, No. 7, July, 1980 pp. 1403–1410

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(Received January 14, 1980)

The granularity of radiographic screen-film systems is assessed by a single number in terms of the entropy method. This method is evaluation on the basis of the precision of measurements, and in the present paper the precision is directly related to radiographic granularity which refers to both RMS granularity (the standard deviation of density fluctuations) and film contrast (the gradient of H & D curve). From the results of entropy calculations, the ability of transmission of radiographic images in two screen-film systems is obtained by comparing the values of transmitted information or the relative efficiency of transmission.

§1. Introduction

The appearance of image noise in the screen-film system of medical radiography, often called radiographic mottle or radiographic granularity, is known to be due to three factors, quantum mottle, structure mottle and film graininess.[1] One method of evaluating image noise quantitatively is measurement of the Wiener spectrum which is essentially a Fourier decomposition of the density fluctuations as measured by a microdensitometer. By using this technique, the noise property of radiographic systems may be evaluated in an analytical way rather than by a single number. On the other hand, one representative measure of granularity by a single number is the standard deviation of density fluctuations of a microdensitometer record, which is also called the root mean square of deviation (RMS).[2] The RMS granularity is equivalent to the root of the integral of the whole Wiener spectrum.[3] Either analytical evaluation or synthetical evaluation by a single number alone is not perfect for image assessment, and use of both methods of evaluation is desirable. However, the RMS granularity may not be considered a sufficient evaluation method of granularity by a single number. It is only a part of the characteristics of the Wiener spectrum, such as the resolving power representing a part of the characteristics of the modulation transfer function (MTF). Hence, a new and synthetical method of evaluation by a single number is now desired.

In 1978, a new and simple method by which the image quality of radiographic images could be synthetically evaluated by a single number in terms of entropy was introduced by Uchida and Tsai, and applied to the problem of evaluating the performance of the process of development.[4] This method of evaluation, termed the entropy method, is considered to be the evaluation on the basis of the precision of measurements. Uchida and Tsai, in 1979, also applied the entropy method to calculation of the relative reliability of MTF's of radiographic screen-film systems.[5] Recently, the authors *et al.* evaluated the performances of several types of thermoluminescence dosimeter (TLD) elements by means of the entropy method.** There was also a report on the evaluation of images for computed tomography scanners using the same method.***

The purpose of the present work is to assess the granularity of radiographic screen-film

*A preliminary report of this work was presented by authors *et al.*: Hoshasenzo Kenkyu (published by Society of Radiation Image Information), **9** (1979) 49 [in Japanese].

**S. Uchida, H. Inatsu and H. Fujita: Proc. 40th Autumn Meeting of the Japan Society of Applied Physics, Hokkaido, September, 1979, 30p-W-1. The paper was submitted to Jpn. J. Appl. Phys. (1979).

***S. Uchida, S. Katsuragawa and T. Sueyoshi: submitted to Jpn. J. Appl. Phys.

systems by a single number in terms of the entropy method. The factors which are closely related to the radiographic granularity assessed by this method will be discussed.

§2. Entropy Method

Shannon's theorems were developed largely within the context of electrical communication channels,[6] and can readily be adapted to any other type of communication system, such as the psychological measurement of the amount of information transmitted by a person.[7,8]

Quantities of the form

$$H=-\sum_{i}^{n} p_i \log_2 p_i \quad \text{[bits]}, \tag{1}$$

where p_i represents the probability of an event, play a central role in information theory as a measure of the average amount of information or uncertainty. H is, in general, called the entropy of a discrete set of probabilities of events $p_1, \cdots, p_i \cdots, p_n$.

Here, we consider the case where the signal is perturbed by noise during transmission in a discrete communication channel for transmitting information.[6] If the entropy of the source or of the input to the channel is $H(x)$, and the entropy of the output of the channel, i.e., the received signal is $H(y)$, then $H(x)=H(y)$ in the noiseless case. In the presence of noise, there are two conditional entropies, $H_y(x)$ and $H_x(y)$, that is, the entropy of the input when the output is known and *vice versa*. $H_y(x)$ is called the equivocation, and $H_x(y)$ ambiguity, sometimes called the channel noise. The joint entropy of input and output is $H(x, y)$. Among these quantities, the relations

$$H(x, y)=H(x)+H_x(y) \tag{2}$$

$$=H(y)+H_y(x) \tag{3}$$

are given. Then, the transmitted information $T(x; y)$ is defined by

$$T(x; y)=H(x)-H_y(x) \tag{4}$$

$$=H(y)-H_x(y) \tag{5}$$

$$=H(x)+H(y)-H(x, y). \tag{6}$$

The following relations are also defined,

$$H(x)+H(y)\geq H(x, y) \tag{7}$$

$$H(x)\geq H_y(x), \quad H(y)\geq H_x(y) \tag{8}$$

$$H(x), \quad H(y)\geq T(x; y). \tag{9}$$

The above situation is illustrated in Fig. 1.[8]

A data matrix of frequency, as shown in Table I, is employed for calculating the above entropies and the transmitted information. The columns and rows of this table represent various discrete inputs and outputs. The capitals X and Y stand for the number of input and output categories, respectively. Note that the subscript i refers to any particular, but unspecified, input while the subscript j refers to any particular, but unspecified, output. The number of times input x_i presented is symbolized by $n_{i\cdot}$ and the frequency, with which the input x_i corresponds to the output y_j, is given by n_{ij}. It is apparent from Table I that

$$\sum_j n_{ij}=n_{i\cdot} \tag{10}$$

$$\sum_i n_{ij}=n_{\cdot j} \tag{11}$$

$$\sum_{ij} n_{ij}=\sum_i n_{i\cdot}=\sum_j n_{\cdot j}=n. \tag{12}$$

Now, three information quantities of $H(x)$,

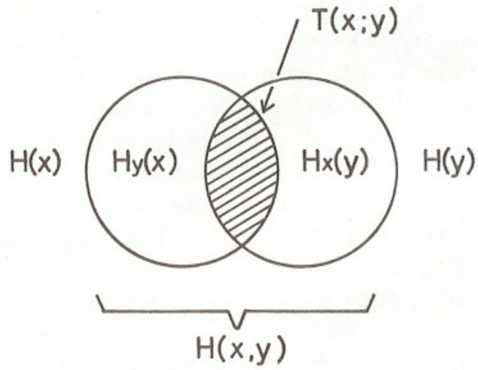

Fig. 1. Venn diagram which represents the relationship between input and output entropies, joint entropy, conditional entropies and the transmitted information. The amounts of each entropy or information are shown by each area in the diagram.

Table I. A data matrix of frequency for Y outputs to X inputs.

Input x

Output y	x_1	$x_2 \cdots x_i \cdots X$	Σ
y_1	n_{11}	$n_{21} \cdots n_{i1} \cdots n_{X1}$	$n_{\cdot 1}$
y_2	n_{12}	$n_{22} \cdots n_{i2} \cdots n_{X2}$	$n_{\cdot 2}$
$\vdots$	$\vdots$	$\vdots \quad \vdots \quad \vdots$	$\vdots$
y_j	n_{1j}	$n_{2j} \cdots n_{ij} \cdots n_{Xj}$	$n_{\cdot j}$
$\vdots$	$\vdots$	$\vdots \quad \vdots \quad \vdots$	$\vdots$
Y	n_{1Y}	$n_{2Y} \cdots n_{iY} \cdots n_{XY}$	$n_{\cdot Y}$
Σ	$n_{1\cdot}$	$n_{2\cdot} \cdots n_{i\cdot} \cdots n_{X\cdot}$	n

$H(y)$ and $H(x, y)$ are calculated by using

$$H(x) = -\sum_i p_{i\cdot} \log_2 p_{i\cdot} \qquad (13)$$

$$H(y) = -\sum_j p_{\cdot j} \log_2 p_{\cdot j} \qquad (14)$$

$$H(x, y) = -\sum_{ij} p_{ij} \log_2 p_{ij}, \qquad (15)$$

where $p_{i\cdot} = n_{i\cdot}/n$, $p_{\cdot j} = n_{\cdot j}/n$ and $p_{ij} = n_{ij}/n$. For simplicity, these equations can be rewritten as

$$H(x) = \log_2 n - \frac{1}{n} \sum_i n_{i\cdot} \log_2 n_{i\cdot} \qquad (16)$$

$$H(y) = \log_2 n - \frac{1}{n} \sum_j n_{\cdot j} \log_2 n_{\cdot j} \qquad (17)$$

$$H(x, y) = \log_2 n - \frac{1}{n} \sum_{ij} n_{ij} \log_2 n_{ij}. \qquad (18)$$

Then the transmitted information $T(x; y)$ is obtained from eq. (6) together with eqs. (16), (17) and (18). As is evident from eqs. (4) and (5), $T(x; y)$ can also be calculated by using the conditional entropies $H_y(x)$ and $H_x(y)$. The relative efficiency of transmission η defined as

$$\eta = \frac{T(x; y)}{H(x)} \times 100\,[\%], \qquad (19)$$

is also obtained.

§3. Experiments and Results

In this paper, the discrete input x_i was considered to be various X-ray exposures obtained by using a 1-mm lucite stepped wedge varying from 0 mm to 5 mm. When the thickness of the stepped wedge increases, the intensity of the transmitted X-ray beam is reduced. The image on the film developed after an X-ray exposure in the screen-film system, therefore, consists of a graduated scale of photographic densities with different values, which correspond to discrete outputs.

Two intensifying screen-film systems which have much the same sensitivity were used. One is LTII-QS screen-film system of medium speed screen-fast speed medical X-ray film, and the other is HS-A system of fast speed screen-standard speed medical X-ray film. The specified exposure factors were kept at 40 kVp, 10 mA, 0.2-second exposure time, and the focus-film distance (FFD) was also kept at 150 cm for LTII-QS system and at 130 cm for HS-A system. This difference of distances is due to the slight difference in sensitivity between the two systems. These systems arranged as in Fig. 2 were exposed simultaneously to X-rays in order to avoid the change of exposure conditions of an X-ray unit. After all films exposed in the screen-film systems were processed by an automatic processor (Sakura QX-200) for 3.5 min at 32±0.3°C, they were measured with both the microdensitometer with a slit area of $8 \times 720\,\mu\text{m}^2$ and the densitometer digitally displayed (Sakura PDA-15) with an aperture size of 0.5 mmϕ. The image density of each step was measured 1000 times for the microdensitometer by sampling intervals of 30 μm, and 30 times for the densitometer by choosing the positions randomly on the image. These various discrete densities of each step were classified by the density level with a density difference $\Delta D = 0.01$ for both measurement cases. The frequency with which the certain density level is shown to each measure was recorded in a frequency matrix of Table II for measurements by microdensitometer and of Table III by densitometer. The columns and rows in these tables represent the thickness and density levels of the lucite steps. By using these frequency matrices, several entropies and the relative efficiency of transmission were calculated, and these values are shown under Tables II and III.

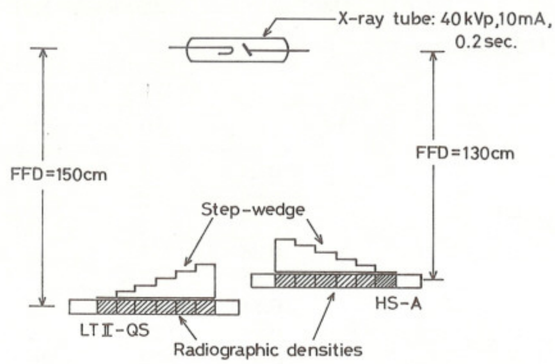

Fig. 2. Schematic diagram of the experimental arrangement.

In our experiments, characteristic curves (H & D curves) and Wiener spectra for each system were also obtained. Figure 3 shows the time-scale H & D curves under the exposure conditions of FFD = 150 cm, 40 kVp and 10 mA. Figure 4 shows Wiener spectra under the microdensitometer conditions of slit area $8 \times 720\,\mu\text{m}^2$, sampling points 1500, sampling intervals 12 μm, and scanning speed 6 μm/s.

Table II. Data matrices of frequency for both systems for measurements by microdensitometer.

HS-A System

Output Y	Input X: 0	1	2	3	4	5	Σ
0.89	1						1
0.88	2						2
0.87	2						2
0.86	7						7
0.85	11						11
0.84	51	2					53
0.83	87	5					92
0.82	122	20					142
0.81	112	33					145
0.80	168	76	5				249
0.79	161	115	15				291
0.78	113	164	26	1			304
0.77	82	167	54	6			309
0.76	46	177	121	9			353
0.75	14	110	150	42	4		320
0.74	17	79	192	70	16	1	375
0.73	2	31	172	104	40	1	350
0.72	2	17	132	139	66	6	362
0.71		4	65	174	105	35	383
0.70			46	186	195	66	493
0.69			14	115	167	105	401
0.68			8	96	210	175	489
0.67				30	87	156	273
0.66				23	66	145	234
0.65				5	33	130	168
0.64					7	103	110
0.63					2	51	53
0.62					2	15	17
0.61						11	11
Σ	1000	1000	1000	1000	1000	1000	6000
Mean	0.799	0.770	0.738	0.708	0.691	0.667	
SD	0.025	0.022	0.022	0.023	0.021	0.023	

SD: Standard Deviation
$H(x)=2.585$ bits, $H(y)=4.330$ bits
$H_y(x)=1.454$ bits, $H_x(y)=3.199$ bits
$H(x, y)=5.784$ bits, $T(x; y)=1.131$ bits
$\eta=43.8\%$

The calculation of Wiener spectra was done by the maximum entropy method (MEM), and was repeated three times and the results were averaged.

§4. Discussion

In the results of entropy calculations for both of the measurements by microdensitometer and densitometer, as shown in Tables II and III, it is apparent that the ability for transmission of radiographic images in HS-A system is superior to that in LTII-QS system by comparing the values of the transmitted information $T(x; y)$ or the relative efficiency of transmission η in each system.

It can be considered that the overlapping degree of adjacent distributions of output data corresponding to each of the different inputs is evaluated by the entropy method.[4] The overlapping degree represents the degree of uncertainty of the data. That is, as the overlapping degree of data distributions becomes larger, the uncertainty of the data obtained increases. It can be considered that there are two factors

Table II. Continued

LTII-QS System

		Input X						
		0	1	2	3	4	5	Σ
	0.81		1					1
	0.80	1	0					1
	0.79	3	7					10
	0.78	8	9	1				18
	0.77	22	17	2	1			42
	0.76	45	31	6	2			84
	0.75	61	56	18	4			139
	0.74	109	102	32	13	3		259
	0.73	108	126	44	25	6	2	311
	0.72	155	159	88	44	27	14	487
	0.71	136	131	101	62	41	11	482
Output Y	0.70	135	132	142	84	50	38	581
	0.69	115	103	137	123	93	76	647
	0.68	57	70	177	179	166	102	751
	0.67	24	28	105	134	148	139	578
	0.66	15	18	74	138	138	165	548
	0.65	6	4	38	78	116	145	387
	0.64		5	27	68	101	130	331
	0.63		1	5	31	59	77	173
	0.62			3	13	34	63	113
	0.61				1	11	20	32
	0.60					5	14	19
	0.59					1	3	4
	0.58					1	1	2
	Σ	1000	1000	1000	1000	1000	1000	6000
	Mean	0.717	0.715	0.692	0.678	0.667	0.658	
	SD	0.026	0.027	0.026	0.026	0.026	0.025	

SD: Standard Deviation
$H(x)=2.585$ bits, $H(y)=3.817$ bits
$H_y(x)=2.175$ bits, $H_x(y)=3.407$ bits
$H(x, y)=5.992$ bits, $T(x; y)=0.410$ bit
$\eta=15.9\%$

which affect the overlapping degree. One is the dispersion of density values measured as output data to each input. The dispersion is closely related to RMS granularity of radiographic screen-film system, especially when the density fluctuations are measured by a microdensitometer. The value of RMS depends on the area of the scanning aperture used to measure the density fluctuations of the film.[9] As the scanning aperture's area increases, the RMS deviation decreases. What is obvious on comparing the SD values in Tables II and III is that the effect of dispersion of density values does not appear largely for the measurement by densitometer, since it has almost the same SD value for each system. It is desirable from these results to measure density fluctuations of the film sample by the microdensitometer. The other is radiographic contrast,[10] which is the difference in density between image areas in the radiograph and depends upon both subject contrast and film contrast. In our experiments, only consideration of film contrast is necessary, because subject contrast is the same for each system by using the same lucite stepped wedge and the same exposure conditions. Film contrast refers to the slope of the characteristic curve of the film. The slope of the H & D curve is called the gradient of the film at that particular density. Film contrast in the screen-film system is mainly affected by the type of film, the processing conditions (development time, temperature

Table III. Data matrices of frequency for both systems for measurements by densitometer.

HS-A System

Output Y \ Input X	0	1	2	3	4	5	Σ
0.58	6						6
0.57	16	1					17
0.56	6	5					11
0.55	2	18					20
0.54		5					5
0.53		1	8				9
0.52			12				12
0.51			10	5			15
0.50				10			10
0.49				14	14	5	33
0.48				1	14	8	23
0.47					2	11	13
0.46						6	6
Σ	30	30	30	30	30	30	180
Mean	0.569	0.550	0.519	0.496	0.484	0.474	
SD	0.008	0.008	0.008	0.008	0.006	0.010	

SD: Standard Deviation
$H(x)=2.585$ bits, $H(y)=3.500$ bits
$H_y(x)=0.706$ bit, $H_x(y)=1.621$ bits
$H(x, y)=4.206$ bits, $T(x; y)=1.879$ bits
$\eta=72.7\%$

LTII-QS System

Output Y \ Input X	0	1	2	3	4	5	Σ
0.55	4						4
0.54	10	6					16
0.53	15	14	3				32
0.52	1	8	8	4			21
0.51		2	16	14	6		38
0.50			2	10	5	3	20
0.49			1	2	17	13	33
0.48					2	13	15
0.47						1	1
Σ	30	30	30	30	30	30	180
Mean	0.536	0.528	0.513	0.507	0.495	0.486	
SD	0.008	0.008	0.009	0.008	0.009	0.007	

SD: Standard Deviation
$H(x)=2.585$ bits, $H(y)=2.852$ bits
$H_y(x)=1.387$ bits, $H_x(y)=1.654$ bits
$H(x, y)=4.239$ bits, $T(x; y)=1.198$ bits
$\eta=46.3\%$

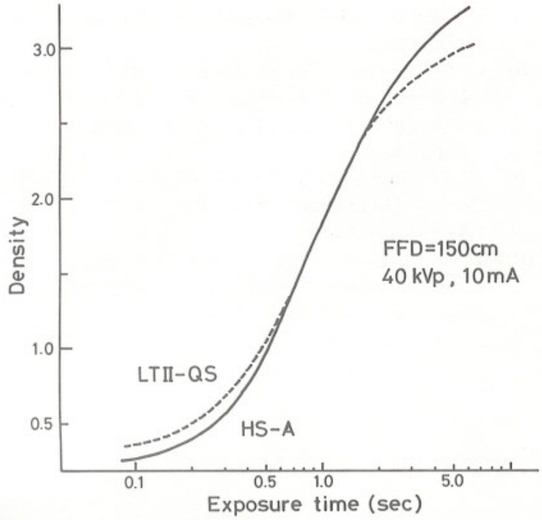

Fig. 3. Time-scale characteristic curves of HS-A and LTII-QS screen-film systems.

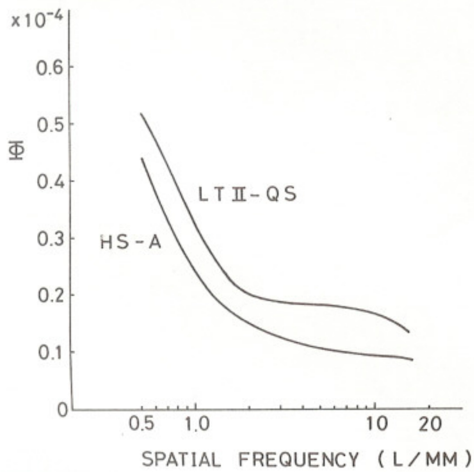

Fig. 4. Measured Wiener spectra of HS-A and LTII-QS systems.

and agitation, etc.), the color of the exposing light from the intensifying screen and the density. The processing conditions for each system are equivalent, and the color of the exposing light is also equivalent because of using the same type of calcium tungstate screens. The density values obtained for both systems are around 0.5 (diffuse density) or 0.7 (specular density) (see Tables II and III). Thus, film contrast depends only upon the type of film here. As can be seen in Fig. 3, the gradient of H & D curve in LTII-QS is smaller than that in HS-A. This effect of the film contrast or the gradient can be seen in Tables II and III on the difference in density between image areas in the stepped wedge.

As discussed above, it is considered that radiographic images in screen-film systems assessed by the entropy method refer to radiographic granularity, in detail, to both RMS granularity and film contrast (the gradient of H & D curve). Radiographic granularity can be represented in the value of transmitted information by a single number by this method. The entropy method is the evaluation on the basis of the precision of measurements,[5] and the precision is directly related to radiographic granularity itself in this paper. The measured Wiener spectra of the two systems used here are shown in Fig. 4. The higher the Wiener value, the greater the radiographic mottle. It is understood that the results of assessment by the entropy method correspond to those by the Wiener spectrum.

§5. Conclusion

The granularity of radiographic screen-film systems is assessed by a single number in terms of the entropy method. From the results of entropy calculations, the ability of transmission of radiographic images in two screen-film systems is obtained by comparing the values of transmitted information or the relative efficiency of transmission. The entropy method is evaluation on the basis of the precision of measurements, and in the present paper the precision is directly related to radiographic granularity itself. It has been shown that the factors which are closely related to the radiographic granularity assessed by this method are both RMS granularity and film contrast.

Acknowledgements

The authors wish to express their great appreciation to Drs. S. Nitta, K. Shimakawa and K. Tanaka of Gifu University for their encouragement and suggestions. Thanks are also due to Messrs. D. Y. Tsai, F. Kumita, H. Sugiyama, M. Fukui, S. Miyata, T. Ishii, H. Mori and H. Abe for their helpful and stimulating discussions. We are indebted to Mr. M. Yokoyama of Seino Transportation Corporation for his valuable advice on the experiments, and to Mrs. V. M. Bowman for her assistance.

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